AVALANCHE GROWTH OF THE SECONDARY RUNAWAY ELECTRON GENERATION

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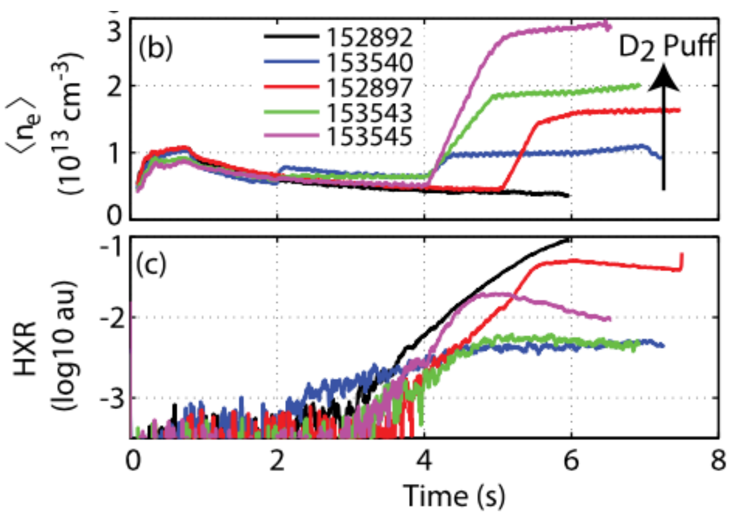
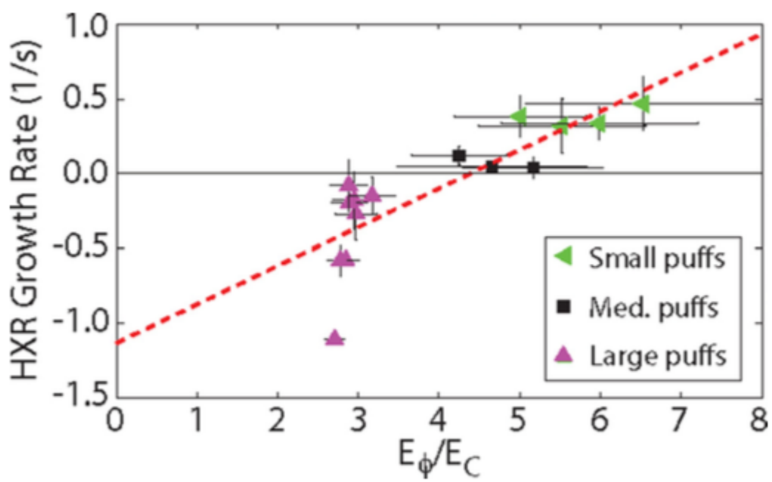
Motivation

- In recent dedicated runaway electron experiments on DIII-D with gas puffing during flat-top, a turning point of the runaway electron HXR signal was observed.
- Critical electric field found to be several times larger than Connor-Hastie E_c.





- Mysterious energy loss mechanisms?

R.S. Granetz et al., Phys. Plasmas **21**, 072506 (2014).
C. Paz-Soldan et al., Physics of Plasmas **21**, 022514 (2014).

Caveats of Rosenbluth-Putvinski

- Rosenbluth-Putvinski's theory predicts E_c (Connor-Hastie critical field) is threshold of secondary generation, and avalanche growth rate (almost) proportional to E/E_c-1.
- Issues with the theory
  - Calculation of secondary generation is based on simplified source term that ignores energy and pitch angle distribution of seed electrons.

$$S = \frac{n_r}{4\pi \ln \Lambda} \delta(\xi - \xi_2) \frac{1}{p^2} \frac{\partial}{\partial p} \left(\frac{1}{1 - \sqrt{1 + p^2}} \right) \qquad \xi_2 = \frac{\sqrt{1+p^2} - 1}{p}$$

  - Radiation effects (synchrotron, bremsstrahlung) ignored in kinetic model.
  - Other kinetic effects (whistler wave, magnetic fluctuation) are also missing.

Outline

- Kinetic model of runaway electrons
  - Synchrotron radiation reaction force
  - Deriving source term for secondary RE generation
- Calculate runaway probability function
  - PDE solving method
  - Critical electric field for growth
- Avalanche growth simulation
  - Growth rate calculation
  - Simulation of gas-puffing case
- Conclusions

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Kinetic model of runaway electrons

- Collisions, radiation effects, and secondary RE generation included in the kinetic equation.

$$\frac{\partial f}{\partial t}+E\{f\}+C\{f\}+\boxed{R\{f\}}=\boxed{S}$$

E: Parallel electric field drive
C: Collision operator
R: Synchrotron radiation reaction force
S: Source term for secondary RE generation

- Collision operator gives correct limits for thermal electrons and relativistic electrons.
- Numerical scheme similar to code CODE.

M. Landreman, A. Stahl, and T. Fülöp, Comp. Phys. Comm. **185**, 847 (2014).

Synchrotron radiation reaction force

- Synchrotron radiation force is important for high energy electrons (comparable to *E* field and collisional drag)

$$\mathbf{F}_S = \frac{2}{3} r_e m_e c^2 \beta^2 \gamma \left\{ \boxed{\frac{\sin^2\theta}{r_g{}^2}\left[(1+p_\perp{}^2)\mathbf{p}_\perp + p_\perp^2 p_\| \hat{b}\right]} + \frac{\beta\gamma^3}{R_0^2}\hat{b} \right\}$$

$$R\{f\} = \nabla \cdot \left(\mathbf{F}_S f\right)$$

- For electrons with γ<100 (most), contribution from the magnetic field curvature is negligible compared to Larmor motion (r_g << R_0).

B. Bernstein and D. C. Baxter, Phys. Fluids **24**, 108 1981.
A. Stahl, M. Landreman, G. Papp, E. Hollmann, and T. Fülöp, Phys. Plasmas **20**, 093302 (2013).

Deriving source term for secondary generation

- We use Møller scattering cross section to get large angle collision scattering probability for relativistic electrons.
- Scattering angle derived from energy and momentum conservation.

$$\cos\theta_\delta = \sqrt{\frac{\gamma_e + 1}{\gamma_e - 1}\frac{\gamma - 1}{\gamma + 1}}.$$

- Source term is integrated from scattering probability and electron distribution function

$$S[f] = \frac{1}{2\pi p^2}\int 2\pi p_e^{\;2} dp_e d\xi_e \hat{S}(p,\xi;p_e,\xi_e) f(p_e,\xi_e).$$

C. Møller, Ann. Phys. (Berlin), 406, 531 (1932)
A.H. Boozer, Phys. Plasmas **22**, 032504 (2015).

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Calculate runaway probability function

- When $E/E_c > 1$, the electron phase space is separated into the runaway region (electron will run away) and lost region (electron will fall back to the thermal population).
- Two methods to study this phase space structure
  - Test particle method - truncate the kinetic equation to make it deterministic, and locate the singular point in phase space.

$$\frac{1}{2}\frac{\partial}{\partial \xi}(1-\xi^2)\frac{\partial f}{\partial \xi} = \frac{\partial}{\partial \xi}(\xi f) + \boxed{\frac{\partial^2}{\partial \xi^2}\left(\frac{1-\xi^2}{2} f\right)}$$

  - Monte-Carlo simulation – Random sampling to obtain runaway probability
- We develop a new method to get runaway probability by solving PDE.

J.R. Martín-Solís, J.D. Alvarez, R. Sánchez, and B. Esposito, Phys. Plasmas **5**, 2370 (1998).
I. Fernández-Gómez, J.R. Martín-Solís, and R. Sánchez, Phys. Plasmas **19**, 102504 (2012).

PDE solving method

- Introduce function P representing the runaway probability

$$P = 1 \quad \text{at high energy boundary}$$
$$P = 0 \quad \text{at low energy boundary}$$

- P is found as a solution to a PDE derived from the kinetic equation. Derivation is similar to first passage problem.

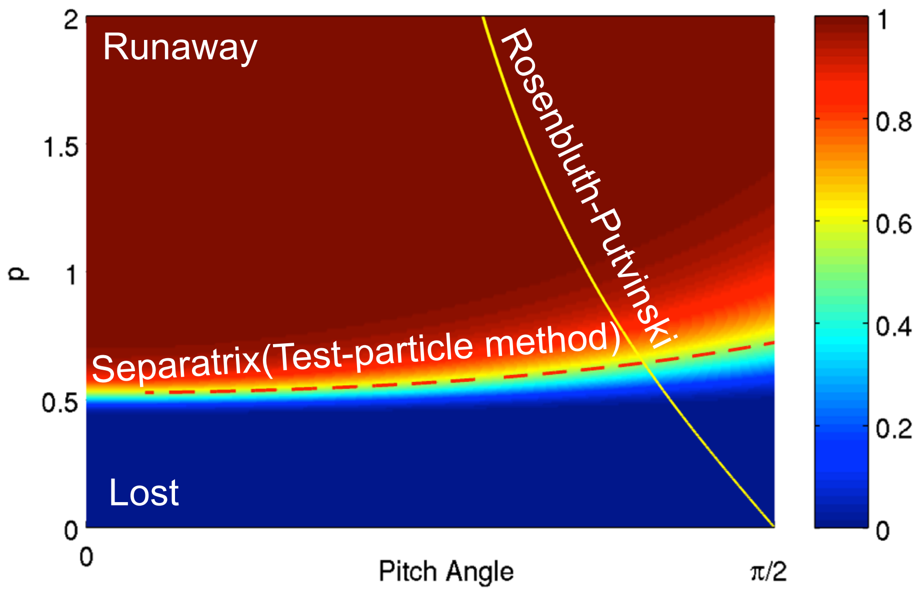
$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial x}\left[v(x) f\right] + \frac{\partial^2}{\partial x^2}\left[D(x) f\right]$$

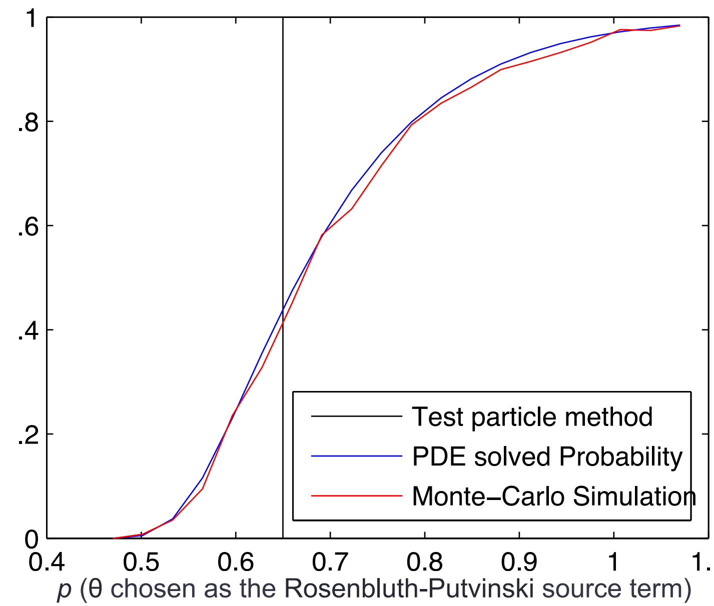
$$\Downarrow$$

$$v(x)\frac{dP(x)}{dx} + D(x)\frac{d^2 P(x)}{dx^2} = 0$$

Adjoint equation of Fokker-Planck equation

Results of runaway probability function

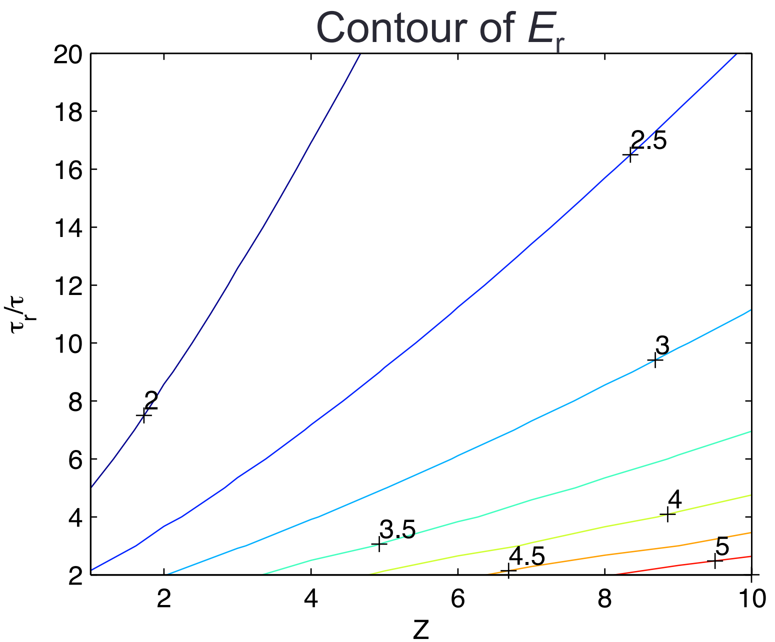


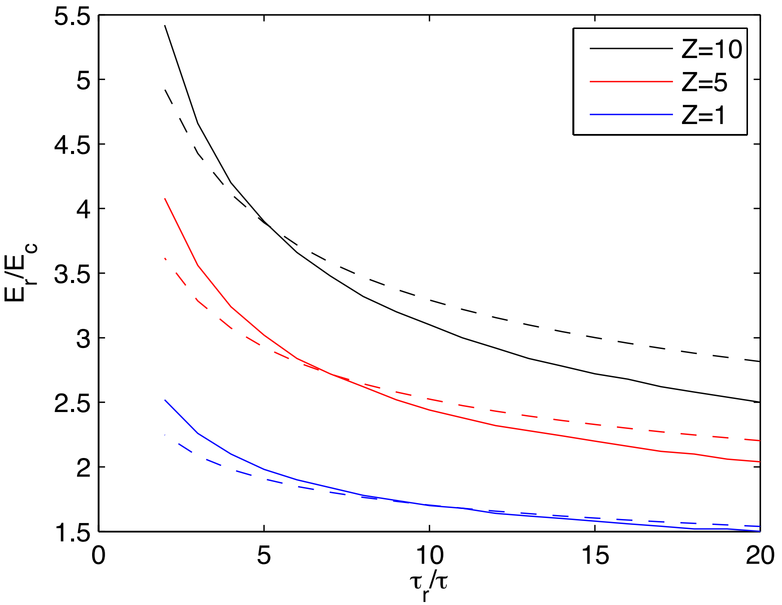


- New method gives smooth probability function rather than separatrix.
- Overcomes caveats of test particle method (truncation & coordinates dependence).
- Agrees well with Monte-Carlo simulation. (Efficiency is better.)

Critical electric field for growth

- In presence of synchrotron radiation force, if E is below a threshold E_r, transition solution is missing, with only a (almost) uniform solution left.
- E_r is the critical electric field for runaway electron growth.





P. Aleynikov and B.N. Breizman, Phys. Rev. Lett. **114**, 155001 (2015).

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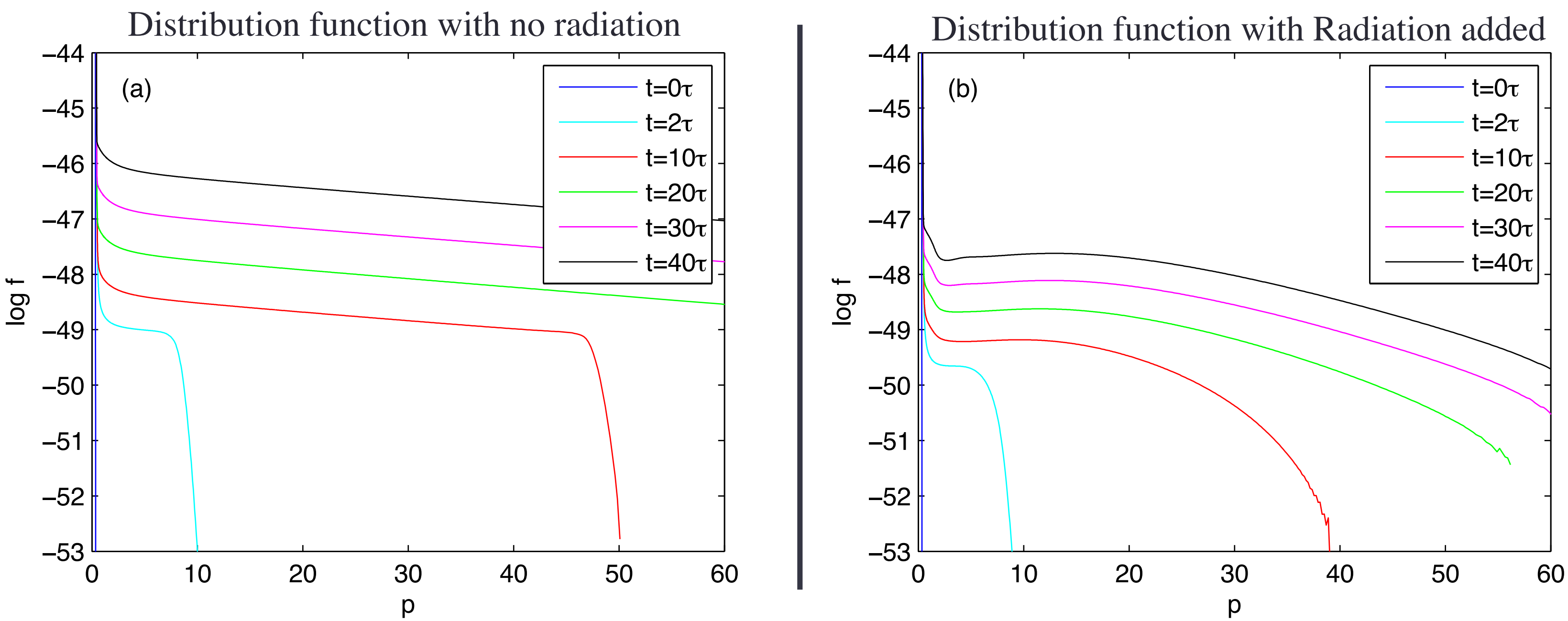
Simulation Result – Avalanche growth

- Time-dependent kinetic equation solved using backward Euler.

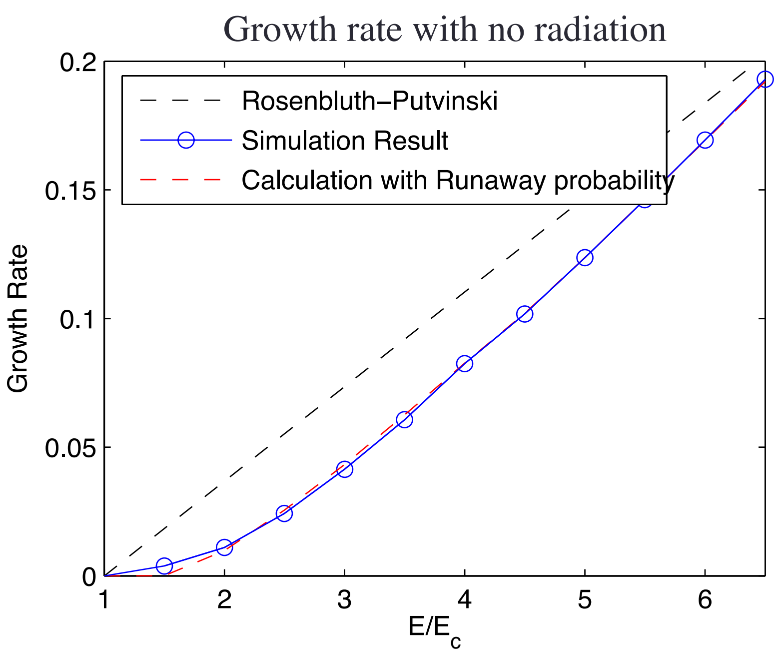
$$\frac{\partial f}{\partial t} + E\{f\} + C\{f\} + R\{f\} = S\{f\}$$

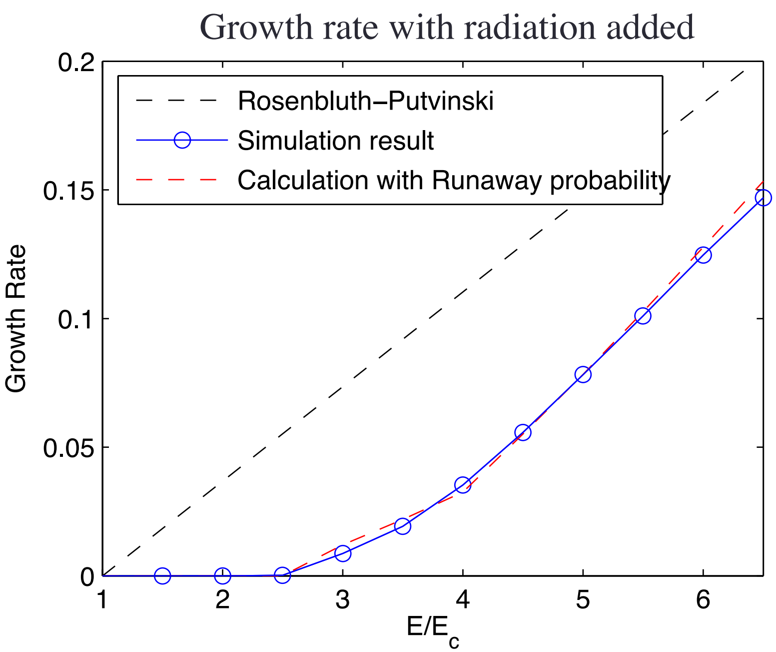


Z=1
τ/τ_r=0.76
(B=3T,
n_e=10^{19}m^3

- With strong radiation, distribution function is non-monotonic.

Avalanche growth rate





Z=1
τ/τ_r=0.76
(B=3T,
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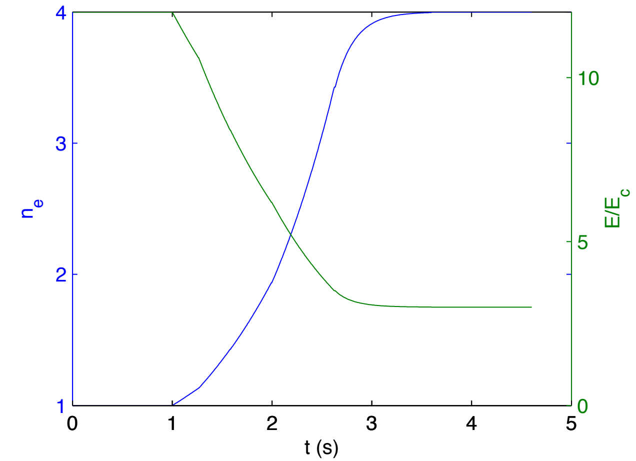
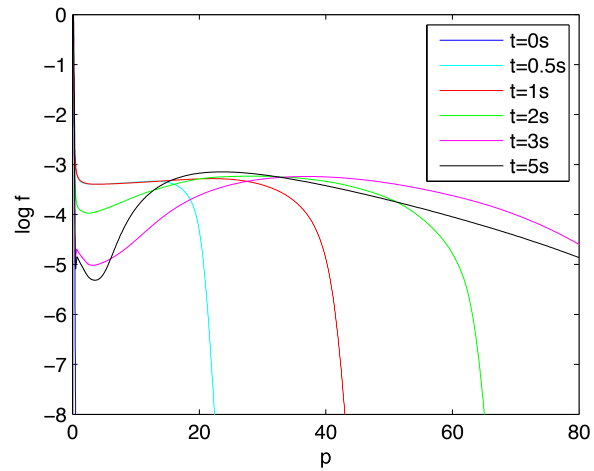
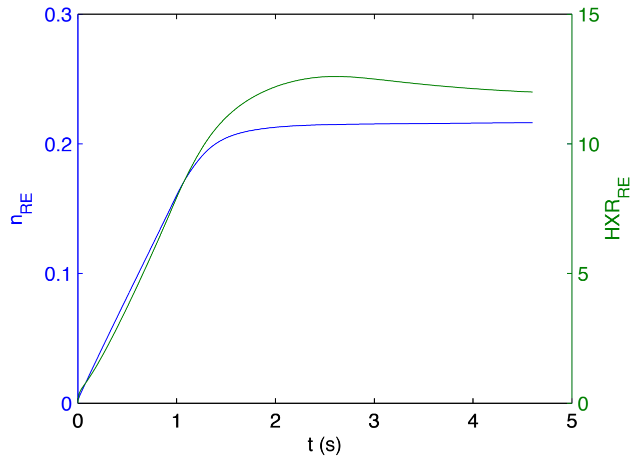
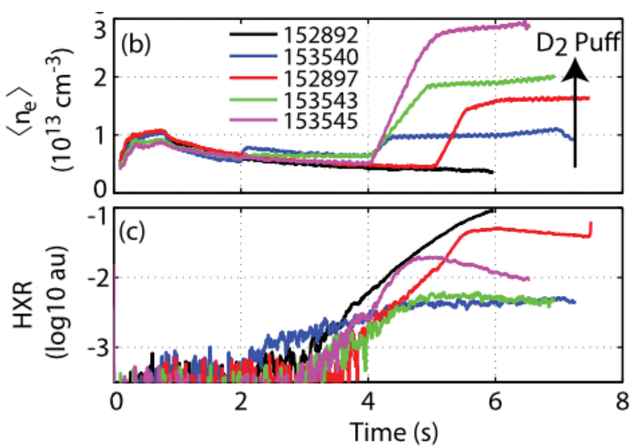
- With synchrotron radiation force added, a new threshold $E_r > E_c$ is observed, below which there is no avalanche growth.

Simulation of gas-puffing case









C. Paz-Soldan et al., Physics of Plasmas **21**, 022514 (2014).

- Three effects after gas puffing: Dreicer loss (loss of low energy electrons), Radiation loss (loss of high energy electrons) and secondary generation.
- HXR signal turning point reflects redistribution of RE energy
- Qualitative agreement with experiment observed. Other loss mechanism not necessary

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Conclusions

- A PDE solving method is developed to calculated the runaway probability function.
- The method can also identify the critical electric field for runaway electron growth.
- In presence of synchrotron radiation reaction and the pitch angle scattering, the threshold electric field for avalanche growth increases from E_c to E_r, which depends on B and Z.
- Simulation of gas-puffing experiment shows qualitatively agreement with the experimental result.
  - Synchrotron radiation
  - Pitch angle scattering Z_{eff}
  - Redistribution of the runaway electron energy

Thank you!

$$\tau = \frac{4\pi\epsilon_0^2 m_e^2 c^3}{e^4 n_e \ln\Lambda}$$

$$\tau_r = \frac{6\pi\epsilon_0 m_e^3 c^3}{e^4 B^2}$$

$$\int f(x,t)P(x)\,dx = \text{const}$$

$$0 = \int \frac{\partial f}{\partial t} P(x)\,dx$$

$$= \int \left\{ -\frac{\partial}{\partial x}\left[v(x) f \right] + \frac{\partial^2}{\partial x^2}\left[D(x) f \right] \right\} P(x)\,dx$$

$$= \int \left\{ v(x)\frac{dP(x)}{dx} + D(x)\frac{d^2 P(x)}{dx^2} \right\} f(x,t)\,dx + \text{Surface term}$$

$$v(x)\frac{dP(x)}{dx} + D(x)\frac{d^2 P(x)}{dx^2} = 0$$

Theoretical estimation of the growth rate

- If a distribution function is given, the growth rate can be calculated using the runaway probability function.

$$\gamma = \frac{1}{n_{re}} \int S(p,\xi) Q(p,\xi) 2\pi p^2 d\xi dp$$

- If a growth rate is given, an approximate distribution can be obtained from the kinetic equation.

$$\Gamma(p) f + \boldsymbol{E}\{f\} + \boldsymbol{C}\{f\} + \boldsymbol{R}\{f\} - \boldsymbol{S}\{f\} = 0$$

- The solution is thus the stationary point.

Next steps

- Study other loss mechanisms, including the bremsstrahlung radiation loss, the magnetic field fluctuation and the whistler wave scattering.
- Study the RE generation and decay for sudden cooling of plasma.

Future work

- Couple the kinetic simulation to MHD code.
- Collaborate on the future DIII-D experiments to study the critical electric field for runaway electron growth and the runaway electron energy distribution.
- Develop more complicated synthetic diagnostics simulations and compare the results with the experiments.